INVESTIGATION OF THE DIFFUSIVE DECAY OF A PLASMA CONTAINED IN A CONDUCTING CYLINDER IN THE PRESENCE OF A MAGNETIC FIELD+)

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Investigation of the diffusive decay of a plasma contained in a conducting cylinder in the presence of a magnetic field.

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Abstract

When a magnetic field is present, the diffusion of a plasma inside a conducting cylinder is not ambipolar. Electric currents are thus associated with the diffusion of particles to the walls. To measure these electric currents directly, a cylinder of mutually insulated segments was used. Measurements were made in a decaying helium plasma with neutral pressures between 0.01 and 0.1 torr and applied magnetic fields between 160 and 2600 gauss. The axial and radial current distributions are obtained. The currents are found to decay exponentially with time, the relative spatial distribution remaining constant. Quantitative analysis of the measured decay times shows, however, that the short-circuit effect suggested by A. SIMON [1)] does not appear. Experimental data obtained for higher magnetic fields may be described in good approximation by a coefficient of transverse diffusion equal to 2.2 times the Bohm diffusion coefficient.

When a plasma is confined by a metal cylinder the diffusion fluxes of electrons and ions to the walls adjust in such a way that a constant potential at the walls results. When a magnetic field is present the ambipolar diffusion mechanism is not compatible with a constant potential along the cylinder surface. This was first stated by A. SIMON [1], who in 1955 criticized the interpretation given by D. BOHM [2] of these diffusion measurements. If the diffusion is not ambipolar, it follows that electric currents are associated with the diffusion process.

This paper gives the results obtained by measuring these currents in a decaying helium plasma.

The apparatus used is shown in Fig. 1. Inside the vacuum container a segmented cylinder is inserted into the homogeneous field of a solenoid. The segmented cylinder is 70 cm in length and 16 cm in diameter. The cylinder surface comprises 7 segments, each end plate consisting of 5 concentric rings. The material used is stainless steel.

Since decay measurements are very sensitive to impurities, special attention had to be paid to the vacuum conditions and to the purity of the gas. The vacuum was generated by an oil diffusion pump shielded with liquid-nitrogen-cooled baffles and with titanium getters evaporated at the top and bottom outside the segmented cylinder (see Fig. 1). After the base pressure of about 1×10^{-8} torr was reached, the inner surface of the segmented cylinder was coated with gold by a removable evaporation source. This was done to minimize the impurities freed by the discharge pulse from the walls. The helium used as working gas first had to pass the getter surface before reaching the discharge volume. Since the measurements were done in a flow system, any impurities that might have been carried in by an insufficiently pure helium would have saturated in the getter. In actual fact, however, the gas was pure enough not to influence the adsorbing rate of the getter.

The plasma inside the segmented cylinder was produced by a pulsed electron beam. In the afterglow following the ionizing pulse the currents to the wall segments were measured. During the plasma decay all the wall segments should be kept at the same constant potential. But, as current measurements require a certain potential drop, we assume that the diffusion process is not influenced when the potential of a single segment is allowed to drift away for about 1 mV because a potential of 1 mV is easily overcome by the thermal energy of the plasma, which is about 0.02 eV.

The circuit used is given in Fig. 2. In particular, the measurement of the current to segment 9 is shown, while the other segments are grounded. Segment 9 is connected to the oscilloscope, which is triggered by the onset of the ionizing pulse. The very good reproducibility allows one decay curve of the current to be obtained by measuring in successive shots with different shunt resistors. Thus the oscillogram in Fig. 2 shows one and the same signal with different amplifications.

In this way, it is possible to measure how the current to a certain wall segment changes during the plasma decay. This time dependence of the wall current is shown in Fig. 3. The abscissa of Fig. 3 gives the time in a linear scale, while the ordinate gives the current to the wall in a logarithmic scale. It can be seen that for time values greater than 20 ms the current changes exponentially with time over three orders of magnitude. The time constant is seen to be the same for a segment in the side wall (ring 9) as for a segment in the end plate (ring 3). The sign of the current to the end plate segment was reversed when plotted in Fig. 3.

The fact that the decay curve shown in Fig. 3 is not exponential for times smaller than 20 ms is due to the presence of higher modes. The plasma production method (see Fig. 1) leads to the situation where at the beginning of the decay most of the plasma is concentrated around the axis of our cylinder. Thus the plasma needs a certain fraction of the decay period to reach the lowest mode. Whether this mode is reached or not may be estimated from the distribution of the currents to the end wall rings (numbers 1 - 5 and 11 - 17 in Fig. 2). The higher the magnetic field, of course, the longer it takes for the ground-mode distribution to be reached. In actual fact, it was found that in a sufficiently late period of decay, when the decay curves of the currents turned out to be exponential, the distribution of the currents to the wall segments was within the measuring accuracy, in agreement with the lowest diffusion mode.

$$n \sim J_o\left(\frac{2.4\ r}{R}\right)\cdot\cos\left(\frac{\pi z}{L}\right)\cdot\exp\left(-\frac{t}{\tau}\right) \qquad (1)$$

In evaluating the measured decay curves, care was taken to use only ground-mode decays.

The diffusive decay in the homogeneous magnetic field is usually described mathematically by the equations (see refs.(1) and (3)):

$$\frac{\partial n_i}{\partial t} = \frac{1}{r}\frac{\partial}{\partial r}\, r\left(D_{i\perp}\frac{\partial n_i}{\partial r} + \mu_{i\perp} n_i \frac{\partial\Phi}{\partial r}\right) + \frac{\partial}{\partial z}\left(D_i \frac{\partial n_i}{\partial z} + \mu_i n_i \frac{\partial\Phi}{\partial z}\right) \qquad (2a)$$

$$\frac{\partial n_e}{\partial t} = \frac{1}{r}\frac{\partial}{\partial r}\, r\left(D_{e\perp}\frac{\partial n_e}{\partial r} + \mu_{e\perp} n_e \frac{\partial\Phi}{\partial r}\right) + \frac{\partial}{\partial z}\left(D_e \frac{\partial n_e}{\partial z} + \mu_e n_e \frac{\partial\Phi}{\partial z}\right) \qquad (2b)$$

where n_i, n_e are the particle densities of the ions and electrons respectively; D_i, $D_{i\perp}$, D_e, $D_{e\perp}$ are the diffusion coefficients parallel and perpendicular to the magnetic field of either particles. The mobilities corresponding are denoted by μ_i, $\mu_{i\perp}$, μ_e, $\mu_{e\perp}$. Φ is the electrical potential. The expressions in brackets are the ion and electron fluxes perpendicular and parallel to the magnetic field. These fluxes are inserted in the particle conservation equations for each species. The equations are coupled by the Poisson equation:

$$\nabla^2\Phi = -\frac{e}{\epsilon_o}(n_i - n_e)\ . \qquad (2c)$$

The coefficients are:

$$D_i = \frac{kT_i}{m_i \nu_{in}}, \qquad D_{i\perp} = \frac{D_i}{1 + \left(\frac{\omega_i}{\nu_{in}}\right)^2}, \qquad (3a)$$

$$D_e = \frac{kT_e}{m_e \nu_{en}}, \qquad D_{e\perp} = \frac{D_e}{1 + \left(\frac{\omega_e}{\nu_{en}}\right)^2}, \qquad (3b)$$

$$\mu = \frac{eD}{kT} \qquad (3c)$$

ν_{in}, ν_{en} : collision frequency of ions and electrons with neutrals,

ω_i , ω_e : cyclotron frequencies.

Since ν_{in}, ν_{en} are proportional to the neutral gas pressure P, equations (2a), (2b) become independent of P if we substitute $\tilde{t} = t/P$ and $\tilde{B} = B/P$ for t and B. Solutions written with $\tilde{t}$ and $\tilde{B}$ thus do not depend on P either. In particular, the decay time $\tilde{\tau} = \tau/P$, where τ is taken from the slopes in the semilogarithmic plots (as in Fig. 3), should be independent of P for constant values of $\tilde{B}$.

Fig. 4 shows that among the measured values there is no systematic deviation from the constancy of τ/P for different pressures, if $\tilde{B}$ is kept constant. The abscissa of Fig. 4 gives in a logarithmic scale the pressure in torr, while the ordinate gives the measured values of $\tilde{\tau}$ in sec/torr. Each symbol belongs to a fixed value of $\tilde{B}$. The fact that τ/P is independent of P allows a mean $\tilde{\tau}$ value to be obtained from the τ's measured at different pressures. Such mean values have the advantage of being less affected by certain pressure-dependent experimental errors, e.g. diffusion cooling of electrons, conversion of atomic to molecular ions, influence of background impurities. In this way, $\tilde{\tau}$ is obtained as a unique function of $\tilde{B}$.

In order to compare the experimental values of $\tilde{\tau}$ with theoretical predictions, we have to enlarge somewhat upon the theory given so far:

The empirical equation (1) is the ground-mode solution of the equation:

$$\frac{\partial n}{\partial t} = D_{\parallel} \frac{\partial^2 n}{\partial z^2} + D_{\perp} \frac{1}{r} \frac{\partial}{\partial r} r \frac{\partial n}{\partial r} \tag{4}$$

for the boundary condition of n = 0 at the surface of a cylinder with the length L and diameter 2R. The decay time τ in equation (1) is then given by

$$\frac{1}{\tau} = \left(\frac{\pi}{L}\right)^2 D_{\parallel} + \left(\frac{2.4}{R}\right)^2 D_{\perp} \tag{5}$$

The theory of ambipolar diffusion and the theories given by SIMON [1] and GOLANT [3] do nothing else but eliminate more or less accurately the mobility terms in the equations (2a), (2b), thus forming from these two equations a single one of type (4). The said theories agree in giving $D_{\parallel} = 2 \cdot D_i$ and

$$\frac{P}{\tau} = \text{const} + \left(\frac{2.4}{R}\right)^2 \cdot P \cdot D_{\perp} \tag{6}$$

As mentioned above, $P \cdot D_{\perp}$ is a function of P/B only (note that this is true also for Bohm diffusion), and we conclude that the most instructive way for presentation of the results is to plot $1/\tilde{\tau}$ as a function of $1/\tilde{B}$. This has been done in Fig. 5. The P/τ values expected from different theories are also given in Fig. 5. The broken lines are calculated with the mobility of A^+ ions in helium in order to show what influence possible impurities [4] would have. From Fig. 5 and equation (6) it is evident that the measured values of $D_{\perp}$ are much smaller than predicted by SIMON and GOLANT. This leads to an interesting interpretation concerning the radial potential distribution:

Measurements were carried out under conditions where we may set $n_i = n_e = n$ and $T_i = T_e = T$. Making the abbrevation:

$$\Psi = \frac{e}{kT} \Phi \tag{7}$$

and using the Einstein relation $\mu = \frac{eD}{kT}$ we can write equations (2a), (2b) in the form

$$\frac{\partial n}{\partial t} = D_i \frac{\partial}{\partial z} \left(\frac{\partial n}{\partial z} + n \frac{\partial \Psi}{\partial z}\right) + D_{i\perp} \frac{1}{r} \frac{\partial}{\partial r} r \left(\frac{\partial n}{\partial r} + n \frac{\partial \Psi}{\partial r}\right) \tag{8a}$$

$$\frac{\partial n}{\partial t} = D_e \frac{\partial}{\partial z} \left(\frac{\partial n}{\partial z} - n \frac{\partial \Psi}{\partial z}\right) + D_{e\perp} \frac{1}{r} \frac{\partial}{\partial r} r \left(\frac{\partial n}{\partial r} - n \frac{\partial \Psi}{\partial r}\right) \tag{8b}$$

Multiplying eq. (8a) by D_e and eq. (8b) by D_i, adding both equations, and making use of $D_i/D_e \ll 1$ leads to

$$\frac{\partial n}{\partial t} = 2\,D_i \frac{\partial^2 n}{\partial z^2} + D_{i\perp} \cdot \frac{1}{r}\frac{\partial}{\partial r} r \left(\frac{\partial n}{\partial r} + n\frac{\partial \Psi}{\partial r}\right) + \frac{D_i D_{e\perp}}{D_e D_{i\perp}} \left(\frac{\partial n}{\partial r} - n\frac{\partial \Psi}{\partial r}\right) \qquad (9)$$

In the experiment under consideration we always had

$$\left(\frac{\omega_e}{\nu_{en}}\right)^2 > 10^4 .$$

From this it follows that

$$0 < \frac{D_i D_{e\perp}}{D_{i\perp} D_e} < 10^{-4} ,$$

and equation (5) may be written

$$\frac{\partial n}{\partial t} = 2\,D_i \frac{\partial^2 n}{\partial z^2} + D_{i\perp} \frac{1}{r}\frac{\partial}{\partial r} r \left(\frac{\partial n}{\partial r} + n\frac{\partial \Psi}{\partial r}\right) . \qquad (10)$$

The difference between the diffusion theories mentioned above results from different assumptions for $\frac{\partial \Psi}{\partial r} = \frac{e}{kT}\frac{\partial \Phi}{\partial r} = -\frac{e}{kT} E_r$. The SIMON [1] short-circuit effect is given by eq. (10) with $\frac{\partial \Psi}{\partial r} = 0$. Fig. 5 shows that this approximation is not very realistic. Therefore, we now turn to the basic assumption made by SIMON, which is

$$E_r = O\left(\frac{\Phi_o}{R}\right) , \qquad E_z = O\left(\frac{\Phi_o}{L}\right) . \qquad (11)$$

Here Φ_o is the potential in the centre of the cylinder and the symbol O( ) means: "of the order of". A similar assumption made by GOLANT is

$$E_r = \xi \frac{1}{n}\frac{\partial n}{\partial r} , \qquad E_z = \xi \frac{1}{n}\frac{\partial n}{\partial z} \qquad (12)$$

with ξ being a constant.

The experimental result as expressed in eq. (1) suggests that eq.(10) be written in the form of eq. (4) and that the mobility term be accounted for by a certain effective diffusion coefficient $D_\perp = D_\perp^{eff}$. Since the diffusion parallel to the magnetic field is essentially ambipolar, we have $\frac{\partial \Psi}{\partial z} < 0$. The assumptions (11) and (12) made for conducting walls therefore give $\frac{\partial\Psi}{\partial r} < 0$. This leads to the following inequality:

$$D_{\perp}^{eff} > D_{i\perp} \tag{13}$$

But, on the other hand, we see from Fig. 5 that the experimentally determined diffusion coefficient $D_{\perp}^{exp}$ always gives

$$D_{\perp}^{exp} < D_{i\perp} \; . \tag{14}$$

We, therefore, conclude from the inequalities (13) and (14) that the short-circuit effect as expressed by eq. (11) and (12) could not be verified and that, on the contrary, we have in actual fact

$$\frac{\partial\Psi}{\partial z} < 0 \quad \text{and} \quad \frac{\partial\Psi}{\partial r} > 0 \tag{15}$$

as it should be in the ambipolar case. (It should be noted that condition (15) cannot be ful~~l~~filled in the whole volume since the wall potential is kept constant.)

We now recall that the short-circuit effect was invented in order to rule out the discomfortable Bohm diffusion. Fig. 5 also gives the P/τ values calculated from $D_{\perp} = D_{\perp}^{Bohm} = \frac{c\ k\ Te}{16\ eB}$ and from $D_{\perp} = 2.2\ D_{\perp}^{Bohm}$, and we see that for magnetic fields higher than about $0.8 \cdot 10^{4}$ gauss/torr experimental values come close to the line calculated with this modified Bohm coefficient.

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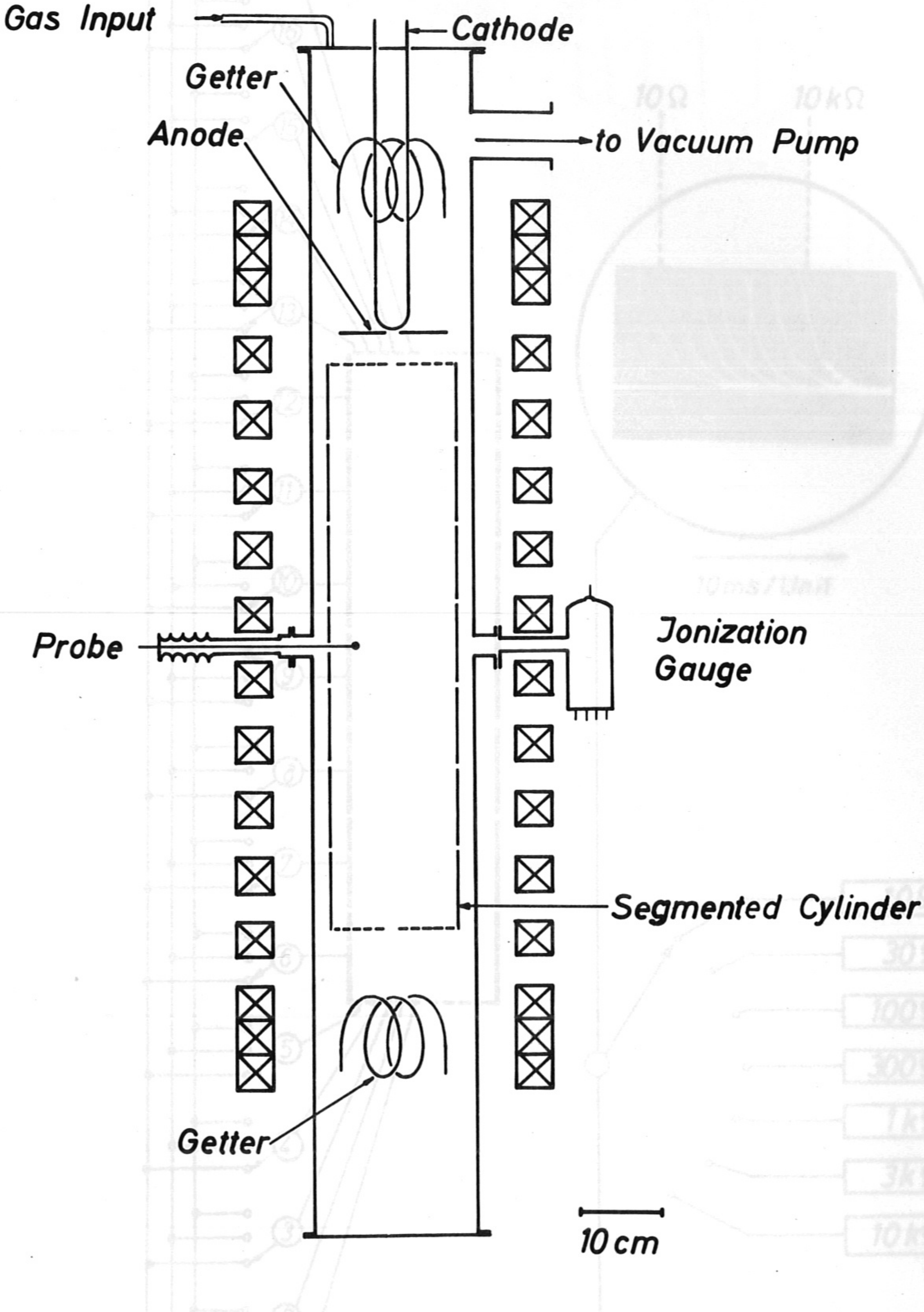


Fig. 1 Apparatus

Typical Operating Data: Neutrals: He at $P = 2 \cdot 10^{-2}$ torr, B-field 10^3 gauss
Pulsed Electron Beam: 30mA, 400V, Duration: 0.5ms
Base Pressure: $2 \cdot 10^{-8}$ torr

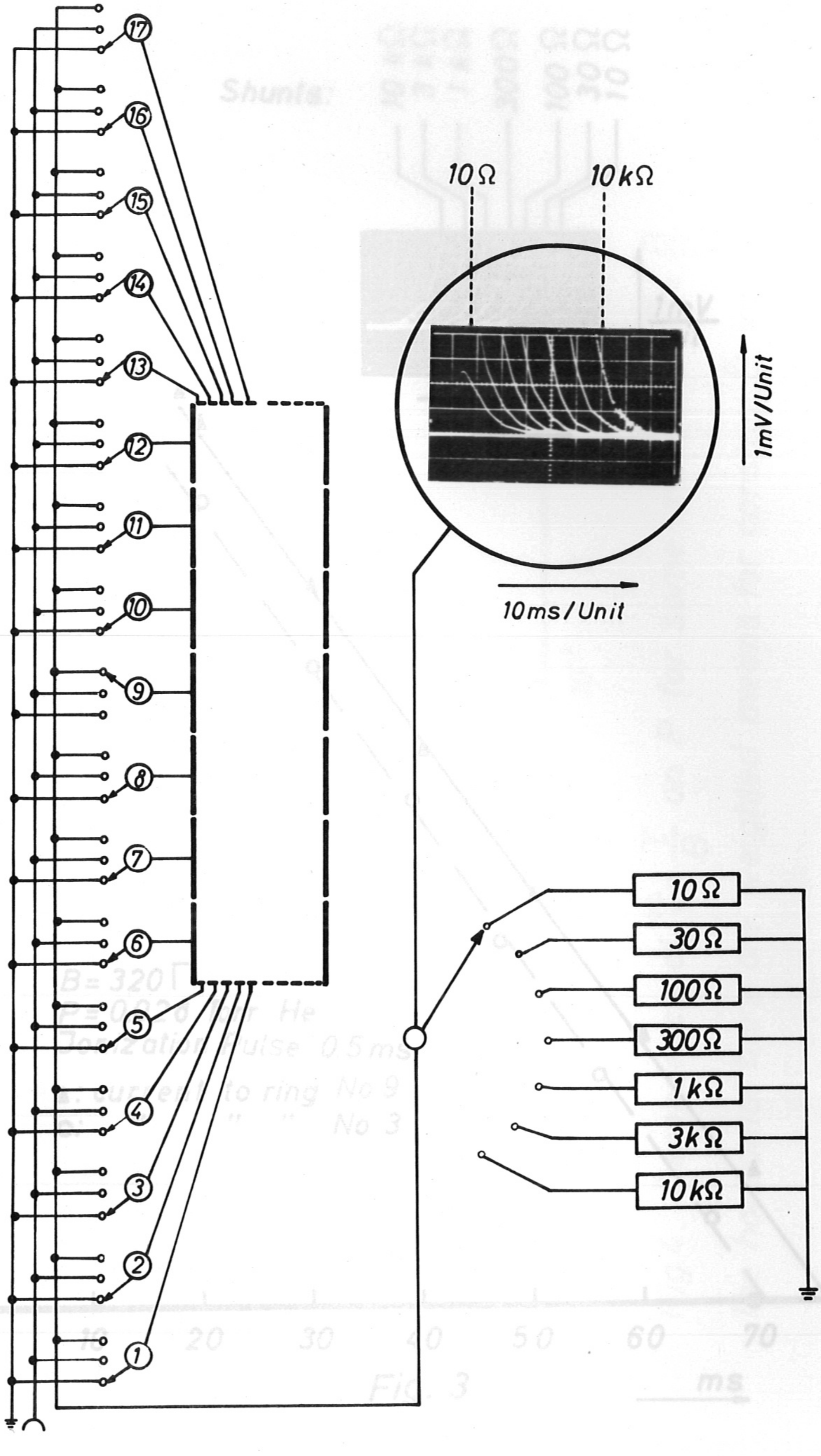
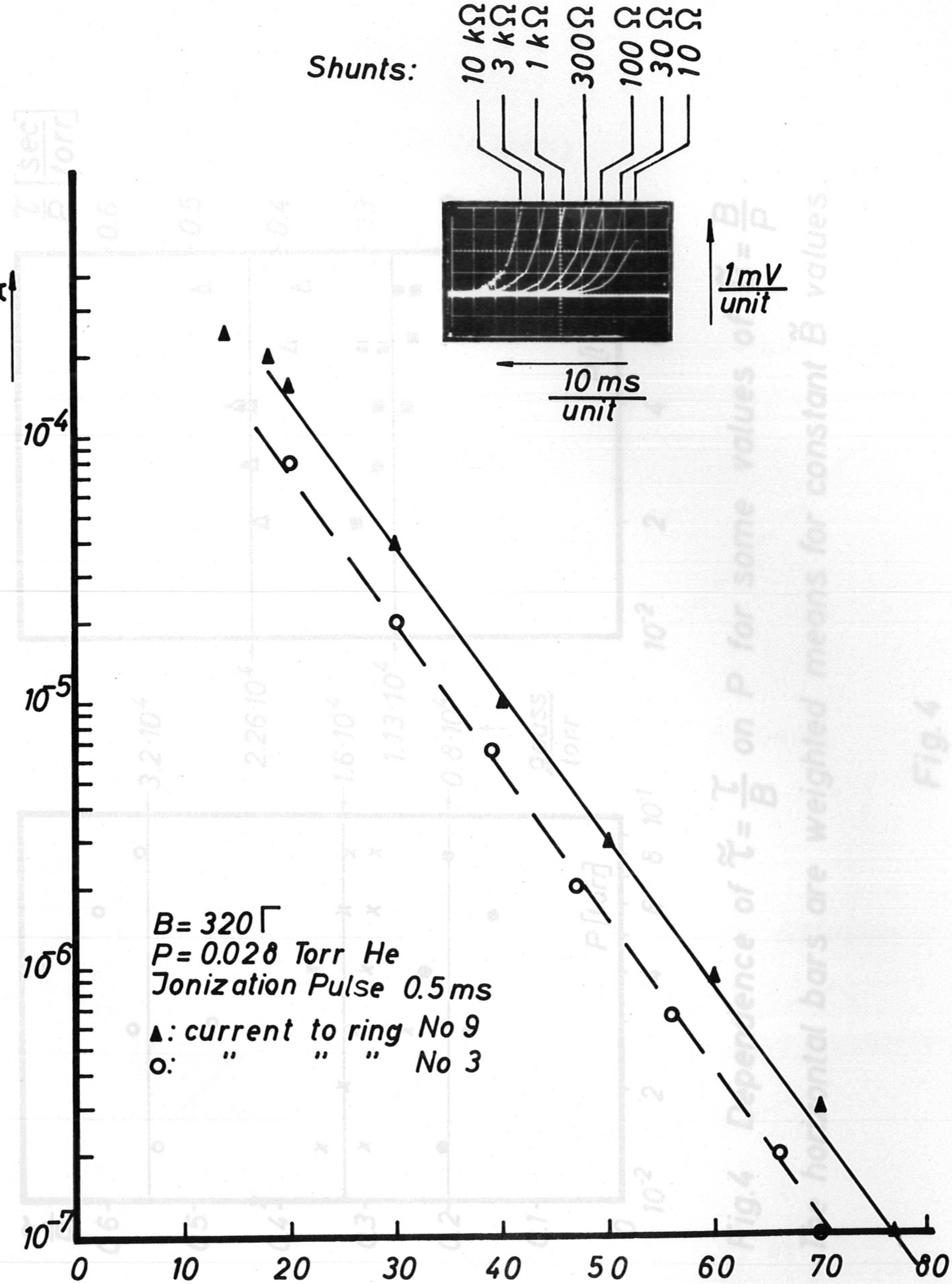

17
16
15
14
13
12
11
10
9
8
7
6
5
4
3
2
1
10Ω
10kΩ
1mV/Unit
10ms/Unit
10Ω
30Ω
100Ω
300Ω
1kΩ
3kΩ
10kΩ

Fig. 2



Shunts:
10 kΩ
3 kΩ
1 kΩ
300 Ω
100 Ω
30 Ω
10 Ω
1mV / unit
10 ms / unit
A
10^{-4}
10^{-5}
10^{-6}
10^{-7}
B= 320 Γ
P= 0.028 Torr He
Jonization Pulse 0.5 ms
▲: current to ring No 9
o: " " " No 3
0
10
20
30
40
50
60
70
80
ms

Fig. 3

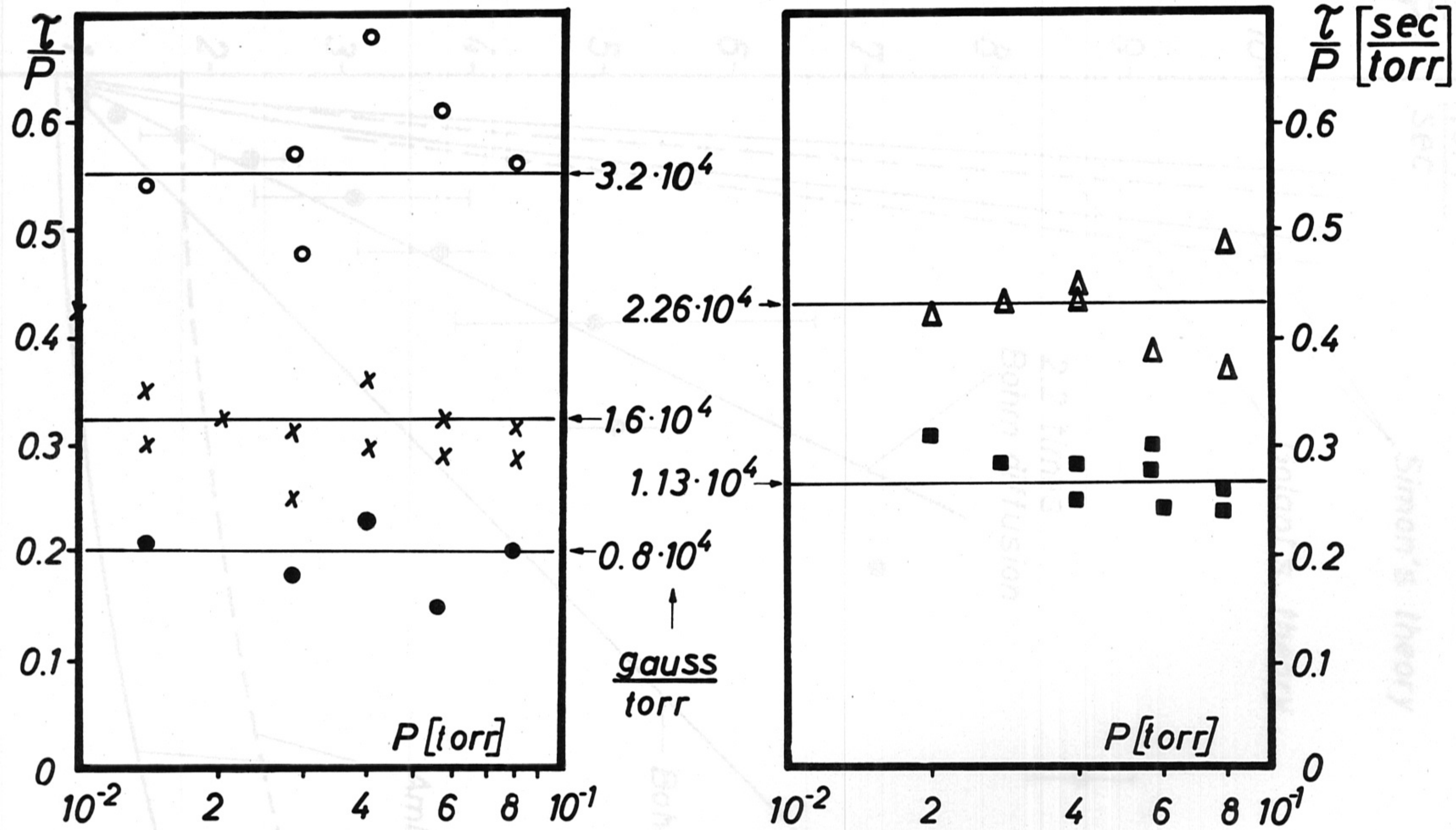
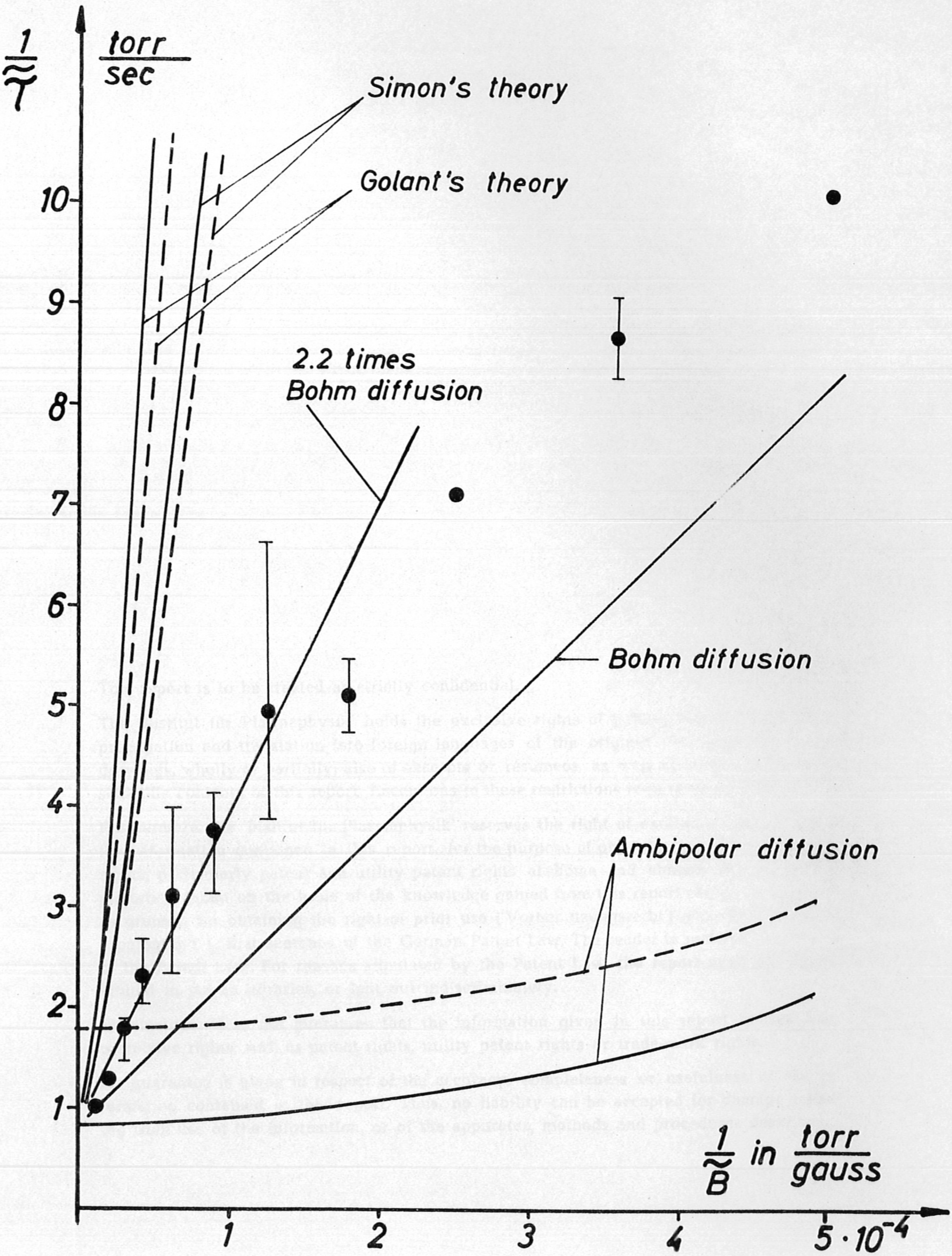


Fig.4 Dependence of $\tilde{\tau} = \frac{\tau}{B}$ on P for some values of $\tilde{B} = \frac{B}{P}$.

The horizontal bars are weighted means for constant $\tilde{B}$ values.

Fig. 4



<u>Fig. 5</u> $\frac{1}{\tilde{\tau}}$ as a function of $\frac{1}{\tilde{B}}$. (Points having no margin of error are taken from single measurements.)